Example of Integral, Taylor, and Kolmogorov Eddy Scale Calculation Using the LIKE Algorithm

Example 3.6B. Consider water flowing in a pipe with D=0.1 m, at an average velocity of 5.5 m/s. The water is at 350 K and 5.7x10^5 Pa. Calculate eddy size, time, velocity, and associated *Re* for the integral, Taylor, and Kolmogorov eddies.

Solution.

Based on the T and P, the water has ν= 3.78x10^{-07} m^2/s. Notice that k and ε are needed, so the LIKE formulas will be used as well.

$$Re_h = \frac{x_{char} u_{char}}{\nu} = \frac{0.1*5.5}{3.78\text{E}-07} = 1.45E06, \ \therefore \text{ turbulent.}$$

$$\ell \approx 0.07 D_h = 0.07*0.1 = 0.007 \text{ m } \left(\text{fast and easy approach}\right)$$

$$I_{meas,\, pipe\ center} = 0.144 Re_h^{-0.146} = 0.144\left(1.45E06\right)^{-0.146} = 0.0181$$

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$$k = \frac{3}{2}\left(\bar{u}I\right)^2 = \frac{3}{2}\left(5.5 * 0.0181\right)^2 = 0.0149 \text{ m}^2/\text{s}^2$$

$$\varepsilon = C_\mu \frac{k^{3/2}}{\ell} = 0.09 * \frac{\left(0.0149\right)^{3/2}}{0.007} = 0.0234 \text{ m}^2/\text{s}^3$$

For the integral eddies,

$$\ell_o = \ell \sim 7.00\text{x}10^{-3} \text{ m } \left(\text{from previous slide}\right)$$

$$\tau_{\ell_o} = C_\mu \frac{k}{\varepsilon} = 0.09\left(\frac{0.0149}{0.0234}\right) = 0.0573 \text{ s}$$

$$\text{v}_{\ell_o} = k^{1/2} = \sqrt{0.0149} = 0.122 \text{ m/s}$$

$$Re_{\ell_o} = C_\mu \frac{k^2}{\varepsilon\nu} = 0.09\left(\frac{0.0149^2}{0.0234x3.78\text{x}10^{-07}}\right) = 2,259$$

Example of Integral, Taylor, and Kolmogorov Eddy Scale Calculation Using the LIKE Algorithm

For the Taylor eddies,

$$\lambda = \left(\frac{10k\nu}{\varepsilon}\right)^{1/2} = \left(\frac{10 * 0.0149 * 3.78\text{x}10^{-07}}{0.0234}\right)^{1/2} = 1.55\text{x}10^{-3}\ \text{m}$$

$$\tau_{\lambda} = \left(\frac{15\nu}{\varepsilon}\right)^{1/2} = \left(\frac{15 * 3.78\text{x}10^{-07}}{0.0234}\right)^{1/2} = 0.0156\ \text{s}$$

$$u_{\lambda} = \left(\frac{2k}{3}\right)^{1/2} = \left(\frac{2 * 0.0149}{3}\right)^{1/2} = 0.0997\ \text{m/s}$$

$$Re_{\lambda} = k\left(\frac{20}{3\varepsilon\nu}\right)^{1/2} = 0.0149\left(\frac{20}{3 * 0.0234 * 3.78\text{x}10^{-07}}\right)^{1/2} = 409$$

Example of Integral, Taylor, and Kolmogorov Eddy Scale Calculation Using the LIKE Algorithm

For the Kolmogorov eddies,

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} = \left[\frac{\left(3.78\text{x}10^{-07}\right)^3}{0.0234}\right]^{1/4} = 3.90\text{x}10^{-05} \text{ m}$$

$$\tau_\eta = \left(\frac{\nu}{\varepsilon}\right)^{1/2} = \left(\frac{3.78\text{x}10^{-07}}{0.0234}\right)^{1/2} = 4.02\text{x}10^{-3} \text{ s}$$

$$\text{v}_\eta = \left(\nu\varepsilon\right)^{1/4} = \left(3.78\text{x}10^{-07} * 0.0234\right)^{1/4} = 9.70\text{x}10^{-3} \text{ m/s}$$

$$Re_\eta = 1$$

Expected Order of Eddy Values

Table 3.6A. Summary of Eddy Scales for Problem 3.6A.

	Integral Eddy	Taylor Eddy	Kolmogorov Eddy
Eddy Size (m)	7.00×10^{-3}	1.55×10^{-3}	3.90×10^{-5}
Eddy Time (s)	0.0573	0.0156	4.02×10^{-3}
Eddy Velocity (m/s)	0.122	0.0997	9.70×10^{-3}
Eddy *Re* (-)	2,259	409	1

Notice:

$$\ell_o > \lambda > \eta$$

$$\tau_{\ell_o} > \tau_\lambda > \tau_\eta$$

$$v_{\ell_o} > v_\lambda > v_\eta$$

$$Re_h > Re_{\ell_o} > Re_\lambda > Re_\eta$$

- **There are many types of *Re*.**
- **Each *Re* has a different physical meaning.**
- **Make sure the correct one is used.**
- **For example, Re_λ = 409 (< 2,200!) for this turbulent problem, while Re_h =1.45×10^6.**