Calculation of k, ε, ω, and ν_t

Example: You need input values for k, ε, ω, and ν_t to run simulations for a rectangular duct under turbulent flow. In addition, what is ν_t / ν? For this situation, H=0.005 m, W=0.025 m, and $\bar{u} = 0.247$ m/s. Re_H =11,000. Hint: try the LIKE algorithm.

Solution:

$$I = 0.16 Re_H^{-1/8} = 0.16\left(11{,}000\right)^{-1/8} = 0.05$$

$$D_H = \frac{4FA}{WP} = \frac{4HW}{2H+2W} = \frac{2HW}{H+W} = \frac{2*0.005*0.025}{0.005+0.025} = 0.00833.$$

$$\ell = 0.07 D_H = 0.07*0.0083 = 0.000583 \text{ m}$$

Now it is straightforward to estimate the values required by the CFD code:

$$k = \frac{3}{2}\left(\bar{u}I\right)^2 = \frac{3}{2}\left(0.247*0.05\right)^2 = 2.29x10^{-4} \text{ m}^2/\text{s}^2$$

$$\varepsilon = C_\mu \frac{k^{3/2}}{\ell} = 0.09\frac{\left(2.29x10^{-4}\right)^{3/2}}{0.000583} = 5.35x10^{-4} \text{ m}^2/\text{s}^3$$

$$\omega = \frac{k^{1/2}}{\ell} = \frac{\left(2.29x10^{-4}\right)^{1/2}}{0.000583} = 26.0 \text{ 1/s}$$

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} = 0.09\frac{\left(2.29x10^{-4}\right)^{2}}{5.35x10^{-4}} = 8.82x10^{-6} \text{m}^2/\text{s}$$

Calculation of k, ε, ω, and ν_t

Get ν from Re:

$$\nu = \frac{D_H \bar{u}}{Re} = \frac{(0.00833)(0.247)}{11{,}000} = 1.87\text{x}10^{-7}\ \text{m}^2/\text{s}.$$

$$\frac{\nu_t}{\nu} = \frac{8.82x10^{-6}}{1.87\text{x}10^{-7}} = 47.2$$

This ratio is consistent with the expectation that ν_t is orders of magnitude larger than ν for turbulent flows.