Reynolds Number
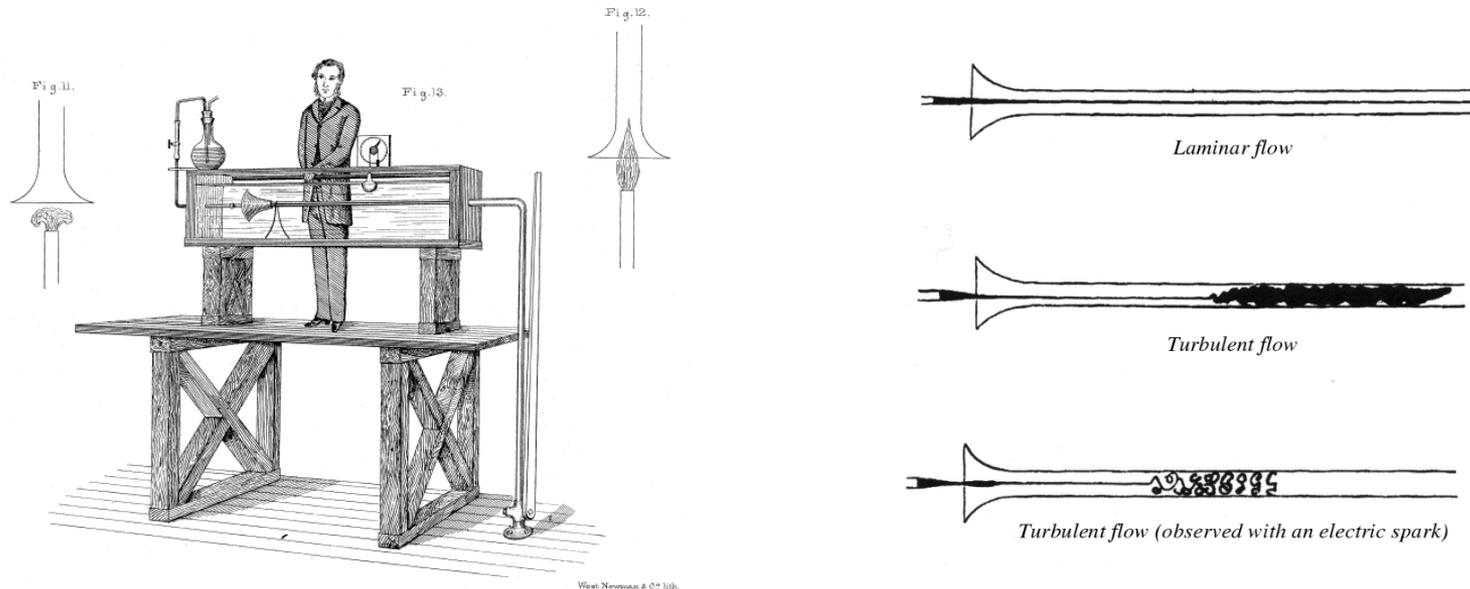
$$\text{Reynolds number} = Re = \frac{x_{char}U_{char}}{\nu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$

x_{char} = characteristic length

U_{char} = characteristic velocity

$$\nu = \frac{\mu}{\rho}$$

- Determines if flow is laminar, transitional, or turbulent.

- Provides clues for turbulence model selection.

Laminar flow

Turbulent flow

Turbulent flow (observed with an electric spark)

Calculating *Re* for Internal Pipe/Duct Flow

Cylindrical pipe with inner diameter D and length L. $x_{char} = D$. $u_{char} =$ average velocity, which can be easily obtained from the mass flow rate,

$$\dot{m} = \rho \overline{V} A$$

or

$$\overline{V} = \frac{\dot{m}}{\rho A} = u_{char}$$

The laminar to turbulent transition occurs at $Re_{crit} = 2,200$ to $2,400$. Wilcox uses 2,300 [Wilcox, 2006].

Internal duct flow (e.g., rectangular pipes; non-circular internal flow), $x_{char} =$ hydraulic diameter (D_h),

$$D_h = \frac{4A}{WP}$$

where

A = flow area and WP = wetted perimeter.

u_{char} = average velocity, and can be obtained from the mass flow rate. The laminar to turbulent transition occurs at $Re_{crit} = 2,200$ to $2,400$.

Calculating *Re* for Flow External to a Cylinder and a Sphere

Flow External to a Cylinder

For flow around (external to) a vertical cylinder, x_{char} = outer diameter D. $u_{char} = U_\infty$ = approach velocity. The laminar to turbulent transition occurs at $Re_{crit} \approx 600$ to 1,000 [Schlichting, 1979]; drag transition $Re_{crit} > 2\text{x}10^5$ to $5\text{x}10^5$ [Schlichting, 1979; Hoerner, 1992].

Flow External to a Sphere

For flow around a sphere, x_{char} = outer diameter D. $u_{char} = U_\infty$ = approach velocity. The laminar to turbulent transition occurs at $Re_{crit} > 300$ [Blevins, 1992]; drag transition $Re_{crit} > 1\text{x}10^5$ to $3\text{x}10^5$ [Schlichting, 1979; Moradian, King, and Cheng, 2008].

Calculating *Re* for Flow External to a Cube ("Square Cylinder") and Jets

Flow External to a Cube or "Square Cylinder"

For flow external to a cube with side S, $x_{char} = S$. $u_{char} = U_\infty =$ approach velocity. The laminar to turbulent transition is in the range of $200 \leq Re_{crit} \leq 260$ [Sohankar, 2006; Bai and Alam, 2018]; drag transition Re_{crit} : none has been observed experimentally up to $Re = 10^7$ [Sohankar, 2006; Bai and Alam, 2018; Khan et al., 2018].

Jet Flow

For jets, $x_{char} =$ jet orifice diameter D. $u_{char} =$ average velocity, which can be obtained from the jet mass flow rate. The jet is fully-turbulent when $Re_{crit} > 3,000$ [McNaughton and Sinclair, 1966].

Calculating *Re* for Couette and Flat Plate Flows

Couette Flow

For Couette flow (specifically, flow between rigid parallel plates separated by a gap H), $x_{char} = H$. u_{char} = plate velocity. [Note: this type of flow is always fully developed (FD)]. The laminar to turbulent transition is at Re_{crit} = 1,500 [Wilcox, 2006].

Flow Over a Flat Plate

For external flow in the x-direction, parallel to a flat plate of length L, the laminar boundary layer $\delta(x)$ grows in the x-direction as [Blasius, 1908],

$$\delta(x)_{lam} \approx \frac{5x}{Re_x^{1/2}}$$

while the turbulent boundary layer grows as

$$\delta(x)_{turb} \approx \frac{0.036x}{Re_x^{1/5}}$$

However, the characteristic length x_{char} for a flat plate is x, so Re reaches a maximum at $x=L$; this type of flow never becomes FD. $u_{char} = U_\infty$ = approach velocity. The laminar to turbulent transition occurs when Re_{crit} ~ 5×10^5 to 10^6 [White, 1991; Schlichting and Gersten, 2000].

Calculating *Re* for an Airfoil or Wing

Flow Over an Airfoil or Wing

For airfoils, x_{char} = chord width (distance from the leading edge to the trailing edge).

For wings, use the mean aerodynamic chord (MAC), which can be approximated as the average of the tip chord and the root chord. Note that more complex MAC approximations include integrals and the center of gravity.

$u_{char} = U_{\infty}$ = relative approach velocity; wind tunnel velocity. The laminar to turbulent transition occurs at $Re_{crit} > 5x10^5$ (approximated as flow over a flat plate).